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RANDOM SEISMIC RESPONSES OF MULTI-SUPPORT STRUCTURES IN EVOLUTIONARY INHOMOGENEOUS RANDOM FIELDS

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SUMMARY

An efficient approach is proposed for analysing the non-stationary random responses of complex structures located in an evolutionary inhomogeneous stochastic field. The approach is a kind of complete CQC method because the cross-correlation terms both between the participant modes and between the ground joint excitations are included in the response calculations. The effect of the loss of coherency between ground joints is also taken into account. For non-proportionally damped structures with many degrees of freedom, the order of the equations of motion can be reduced by using only real modes while structural non-stationary random responses can still be computed conveniently and accurately.

KEY WORDS: structures; response; seismic; random; multi-support; earthquake; evolutionary; inhomogeneous; multi-excitation

INTRODUCTION

Multi-support structures are being widely used nowadays, particularly in major public facilities. An important problem in the design of such structures is their reliability under seismic excitations, and it is recognized that for such long-span structures, asynchronous ground joint excitations must be taken into account. During the last fifteen years, the seismic response analysis of multi-support structures has received much attention.[1–9] Typically, for the case of stationary multi-support excitations, Kiureghian and Neuenhofer[7] and Zavoni and Vanmarcke[8] published their new methods in the last four years. Lin *et al.*[4,9,10] proposed a new fast-CQC method which is very efficient. For the much more difficult non-stationary seismic analysis of multi-support structures, Perotti[11] and Harichandran[12] proposed some schemes, which provide pioneering and instructive explorations.

Recently, the present authors proposed an efficient pseudo-excitation method to compute non-stationary responses of structures subjected to uniformly modulated evolutionary single-point seismic excitation.[13] In the present paper, this method is extended to compute the non-stationary seismic responses of multi-support structures, for which the wave passage (or travelling wave) effect and the loss of coherency between ground excitations are both included. This method is readily coded for very efficient computation. It is widely known that the abbreviation CQC (Complete Quadratic Combination) was first used by Wilson and Kiureghian (see Reference 6 in Reference 9) as a modal combination rule based on random vibration theory, its essence being that the cross-correlation terms of the participant modes are all included in the response analysis. This

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abbreviation has also been used in Reference 9 to characterize the pseudo-excitation algorithm because it is impossible for this random vibration algorithm to neglect the cross-correlation terms between the participant modes. In the present paper, the pseudo-excitation method has been further extended so that the cross-correlation terms between the ground joint excitations are also included. Hence, it can be regarded as another sort of Complete Quadratic Combination approach. In addition, non-proportionally damped structures are dealt with easily and accurately. The method enables analyses of complex structural models with many ground joints to be completed very conveniently, e.g. on ordinary workstations or even IBM/486 personal computers. Numerical analyses based on several multi-point excitation models[5,6] are presented.

EQUATIONS OF MOTION OF MULTI-SUPPORT STRUCTURES

For a long-span structure, the total displacement vector in the global co-ordinate system can be partitioned as

$$\mathbf{X} = \begin{Bmatrix} \mathbf{X}_{\mathrm{s}} \\ \mathbf{X}_{\mathrm{m}} \end{Bmatrix} \tag{1}$$

in which $\mathbf{X}_{\mathrm{m}}$ is the enforced displacement vector of the N ground joints, and $\mathbf{X}_{\mathrm{s}}$ is the displacement vector of the remaining structural nodes, which can be further decomposed into

$$\mathbf{X}_{\mathrm{s}} = \mathbf{Y}_{\mathrm{s}} + \mathbf{Y}_{\mathrm{r}} \tag{2}$$

where $\mathbf{Y}_{\mathrm{s}}$ is the quasi-static displacement vector

$$\mathbf{Y}_{\mathrm{s}} = -\mathbf{K}_{\mathrm{ss}}^{-1}\mathbf{K}_{\mathrm{sm}}\mathbf{X}_{\mathrm{m}} \tag{3}$$

while the dynamic displacement vector $\mathbf{Y}_{\mathrm{r}}$ satisfies the equation[3,9]

$$\mathbf{M}_{\mathrm{ss}}\ddot{\mathbf{Y}}_{\mathrm{r}} + \mathbf{C}_{\mathrm{ss}}\dot{\mathbf{Y}}_{\mathrm{r}} + \mathbf{K}_{\mathrm{ss}}\mathbf{Y}_{\mathrm{r}} = -\mathbf{M}_{\mathrm{ss}}\ddot{\mathbf{Y}}_{\mathrm{s}} \tag{4}$$

and the subscripts s and m denote partitioning of **K**, **M** and **C** compatibly with equation (1). Equation (4) has been strictly proved[3] under the assumption $\mathbf{M}_{\mathrm{sm}} = \mathbf{0}$, i.e. for lumped mass systems.

STRUCTURAL RESPONSE TO SINGLE-POINT EVOLUTIONARY RANDOM EXCITATION

Consider an evolutionary random excitation exerted on an initially static structure which takes the form

$$f(t) = g(t)x(t) \tag{5}$$

in which $g(t)$ is a specified modulation (or envelope) function, and $x(t)$ is a zero-mean-valued stationary random process with its PSD $S_{xx}(\omega)$ given. It has been proved that constituting the pseudo-excitation[13]

$$f(t) = \sqrt{S_{xx}(\omega)}\,g(t)\exp(\mathrm{i}\omega t) \tag{6}$$

results in the fact that the time-dependent PSD matrices of any two arbitrarily chosen response vectors **y** and **z**, must be

$$\mathbf{S}_{yy}(\omega, t) = \mathbf{y}(\omega, t)^{*}\mathbf{y}(\omega, t)^{\mathrm{T}}, \qquad \mathbf{S}_{yz}(\omega, t) = \mathbf{y}(\omega, t)^{*}\mathbf{z}(\omega, t)^{\mathrm{T}} \tag{7}$$

where the asterisk denotes complex conjugate. Thus, various time-dependent PSD can be calculated from deterministic dynamic analyses, see beneath equation (24) below. In general, the analysis of pseudo-responses can be executed in the time domain, e.g. by using Duhamel integration or Newmark's method. However, when the envelope function $g(t)$ is simple, it can sometimes also be executed in the frequency domain by using analytical methods.[13]

NON-STATIONARY RANDOM SEISMIC RESPONSES OF STRUCTURES SUBJECTED TO MULTI-PHASE SEISMIC EXCITATIONS

Consider an initially static structure with N ground joints which have displacements $\mathbf{X}_{\mathrm{m}}$ in the absolute co-ordinate system. Suppose that the ground acceleration at the origin of this co-ordinate system has the evolutionary form

$$\ddot{X}_{\mathrm{m}}(t) = g(t)x(t) \tag{8}$$

and that the time lag between the jth ground joint and the origin is T_j ($j = 1, 2, \ldots, N$), with $T_j \geqslant 0$ assumed without loss of generality. The acceleration vector for the N ground joints is then

$$\ddot{\mathbf{X}}_{\mathrm{m}}(t) = \mathbf{G}(t)\mathbf{X}(t) = \begin{bmatrix} g(t-T_1) & & & \\ & g(t-T_2) & & \\ & & \ddots & \\ & & & g(t-T_N) \end{bmatrix} \begin{Bmatrix} x(t-T_1) \\ x(t-T_2) \\ \vdots \\ x(t-T_N) \end{Bmatrix} \tag{9}$$

where $\mathbf{G}(t)$ is the diagonal time-shifted modulation function matrix and $\mathbf{X}(t)$ is the time-shifted stationary random process vector. An arbitrary response vector $\mathbf{z}(t)$ is given, in terms of its impulse response functions $h(t)$, by Duhamel integration as

$$\mathbf{z}(t) = \int_0^t \mathbf{h}(t-\tau)\ddot{\mathbf{X}}_{\mathrm{m}}(\tau)\,\mathrm{d}\tau = \int_0^t \mathbf{h}(t-\tau)\mathbf{G}(\tau)\mathbf{X}(\tau)\,\mathrm{d}\tau \tag{10}$$

If $E[\cdots]$ denotes taking the average over the ensemble of realizations, the cross-correlation matrix between two arbitrary response vectors $\mathbf{z}_k(t_k)$ and $\mathbf{z}_l(t_l)$ is

$$\begin{aligned} \mathbf{R}_{z_k z_l}(t_k, t_l) &= E[\mathbf{z}_k(t_k)\mathbf{z}_l(t_l)^{\mathrm{T}}] \\ &= \int_0^{t_k}\int_0^{t_l} \mathbf{h}_k(t_k-\tau_k)\mathbf{G}(\tau_k)E[\mathbf{X}(\tau_k)\mathbf{X}(\tau_l)^{\mathrm{T}}]\mathbf{G}(\tau_l)^{\mathrm{T}}\mathbf{h}_l(t_l-\tau_l)^{\mathrm{T}}\,\mathrm{d}\tau_k\,\mathrm{d}\tau_l \end{aligned} \tag{11}$$

Now, let

$$\begin{aligned} [\phi_{kl}(\tau_k, \tau_l) &\equiv \mathbf{E}[\{\mathbf{X}(\tau_k)\}\{\mathbf{X}(\tau_l)\}^{\mathrm{T}}] \\ &= E\left[\begin{Bmatrix} x(\tau_k-T_1) \\ x(\tau_k-T_2) \\ \vdots \\ x(\tau_k-T_N) \end{Bmatrix} \{x(\tau_l-T_1), x(\tau_l-T_2), \ldots, x(\tau_l-T_N)\}\right] \\ &= E\begin{bmatrix} x(\tau_k-T_1)x(\tau_l-T_1) & x(\tau_k-T_1)x(\tau_l-T_2) & \cdots & x(\tau_k-T_1)x(\tau_l-T_N) \\ x(\tau_k-T_2)x(\tau_l-T_1) & x(\tau_k-T_2)x(\tau_l-T_2) & \cdots & x(\tau_k-T_2)x(\tau_l-T_N) \\ \vdots & \vdots & & \vdots \\ x(\tau_k-T_N)x(\tau_l-T_1) & x(\tau_k-T_N)x(\tau_l-T_2) & \cdots & x(\tau_k-T_N)x(\tau_l-T_N) \end{bmatrix} \end{aligned} \tag{12}$$

Since $x(t)$ is a stationary random process with its PSD given, the above equation can be expressed, by letting $\tau = \tau_l - \tau_k$ and using the Wiener–Khinchene relation, as

$$[\phi_{kl}(\tau_k, \tau_l)] = [\phi_{kl}(\tau)]$$

$$= \int_{-\infty}^{\infty} \begin{bmatrix} 1 & \exp(\mathrm{i}\omega(T_1 - T_2)) & \cdots & \exp(\mathrm{i}\omega(T_1 - T_N)) \\ \exp(\mathrm{i}\omega(T_2 - T_1)) & 1 & \cdots & \exp(\mathrm{i}\omega(T_2 - T_N)) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(\mathrm{i}\omega(T_N - T_1)) & \exp(\mathrm{i}\omega(T_N - T_2)) & \cdots & 1 \end{bmatrix} \mathrm{e}^{\mathrm{i}\omega\tau} S_{xx}(\omega)\,\mathrm{d}\omega$$

$$= \int_{-\infty}^{\infty} \mathbf{V}^* \mathbf{R}_0 \mathbf{V}^{\mathrm{T}} \exp(\mathrm{i}\omega\tau) S_{xx}(\omega)\,\mathrm{d}\omega \tag{13}$$

in which

$$\mathbf{V} = \mathrm{diag}[\exp(-\mathrm{i}\omega T_1, \exp(-\mathrm{i}\omega T_2), \ldots, \exp(-\mathrm{i}\omega T_N)]^{\mathrm{T}} \tag{14}$$

$$\mathbf{R}_0 = \mathbf{q}_0 \mathbf{q}_0^{\mathrm{T}} \tag{15}$$

$\mathbf{q}_0$ and $\mathbf{R}_0$ are the vector and square matrix with all of their elements equal to unity. Equations (11)–(15) now lead to[9]

$$\mathbf{R}_{z_k z_l}(t_k, t_l) = \int_{-\infty}^{\infty} \mathbf{I}_k^* \mathbf{I}_l^{\mathrm{T}} S_{xx}(\omega)\,\mathrm{d}\omega \tag{16}$$

in which $\mathbf{I}_n (n = k \text{ or } l)$ is given by

$$\mathbf{I}_n = \int_0^{t_n} \mathbf{h}_n(t_n - \tau_n) \mathbf{G}(\tau_n) \mathbf{V} \mathbf{q}_0 \exp(\mathrm{i}\omega\tau_n)\,\mathrm{d}\tau_n \tag{17}$$

The integrand of equation (16) is the cross-PSD matrix between $\mathbf{z}_k(t_k)$ and $\mathbf{z}_l(t_l)$, i.e.

$$\mathbf{S}_{z_k z_l}(t_k, t_l, \omega) = \mathbf{I}_k^* \mathbf{I}_l^{\mathrm{T}} S_{xx}(\omega) \tag{18}$$

It is known from equation (17) that $\mathbf{I}_k(t_k)$ is the response caused by the deterministic loading $\mathbf{G}(t)\mathbf{V}\mathbf{q}_0 \exp(\mathrm{i}\omega t)$ at time $t = t_k$. Hence, the pseudo-excitation vector for this problem can be expressed as

$$\ddot{\mathbf{X}}_{\mathrm{m}}(t, \omega) = \mathbf{G}(t) \mathbf{V} \mathbf{q}_0 \exp(\mathrm{i}\omega t) \sqrt{S_{xx}(\omega)} \tag{19}$$

The responses caused by this deterministic pseudo-loading would be

$$\mathbf{z}_k(t_k, \omega) = \sqrt{S_{xx}(\omega)}\,\mathbf{I}_k(t_k), \qquad \mathbf{z}_l(t_l, \omega) = \sqrt{S_{xx}(\omega)}\,\mathbf{I}_l(t_l) \tag{20}$$

Therefore,

$$\mathbf{z}_k(t_k, \omega)^* \mathbf{z}_l(t_l, \omega)^{\mathrm{T}} = \mathbf{I}_k^* \mathbf{I}_l^{\mathrm{T}} S_{xx}(\omega) \tag{21}$$

Comparing equations (18) and (21) gives

$$\mathbf{S}_{z_k z_l}(t_k, t_l, \omega) = \mathbf{z}_k(t_k, \omega)^* \mathbf{z}_l(t_l, \omega)^{\mathrm{T}} \tag{22}$$

By letting $k = l$ and $t_k = t_l = t$, equation (22) produces the PSD matrix of a response vector $\mathbf{z}$ relative to itself as

$$\mathbf{S}_{zz}(t, \omega) = \mathbf{z}(t, \omega)^* \mathbf{z}(t, \omega)^{\mathrm{T}} \tag{23}$$

Obviously, using equations (19) and (15), the PSD matrix of $\ddot{\mathbf{X}}_{\mathrm{m}}(t, \omega)$ would be

$$\mathbf{S}_{\ddot{x}_{\mathrm{m}}\ddot{x}_{\mathrm{m}}}(t, \omega) = \ddot{\mathbf{X}}_{\mathrm{m}}^* \ddot{\mathbf{X}}_{\mathrm{m}}^{\mathrm{T}} = S_{xx}(\omega) \mathbf{G}^* \mathbf{V}^* \mathbf{q}_0^* \mathbf{q}_0^{\mathrm{T}} \mathbf{V}^{\mathrm{T}} \mathbf{G}^{\mathrm{T}} = S_{xx}(\omega) \mathbf{G} \mathbf{V}^* \mathbf{R}_0 \mathbf{V} \mathbf{G} \tag{24}$$

The above procedure should be executed for a series of discrete frequencies. For each of them, the deterministic step-by-step integration of equation (4) must be executed twice (corresponding to the real and imaginary parts of its right-hand side), which constitutes the major part of the computational effort. The time-dependent variance of an arbitrary element v of vector $\mathbf{z}$ can then be calculated from

$$\sigma_v^2(t) = 2\int_0^{\infty} S_{vv}(t, \omega)\,\mathrm{d}\omega \tag{25}$$

To summarize, it can be seen that such non-stationary random responses can be conveniently analysed by solving the equations of motion by introducing a deterministic pseudo-ground acceleration vector.

The modulation function $g(t)$ of equation (8) is a slowly varying time-dependent function, so that the variation of $\ddot{\mathbf{X}}_{\mathrm{m}}(t)$ within a very short time interval is caused mainly by the variation of $x(t)$. Integrating equation (19) twice gives the pseudo-ground displacement vector

$$\mathbf{X}_{\mathrm{m}}(t, \omega) = -\frac{1}{\omega^2}\ddot{\mathbf{X}}_{\mathrm{m}}(t, \omega) \tag{26}$$

Substituting equations (19) and (26) into equations (1)–(4) gives the pseudo-responses $\mathbf{Y}_{\mathrm{s}}$, $\mathbf{Y}_{\mathrm{r}}$ and $\mathbf{X}_{\mathrm{s}}$, from which all required internal force responses, etc., can easily be obtained. Furthermore, the corresponding PSD matrices can be computed by using equations (22) and (23).

To reduce the size of the problem, the mode-superposition scheme can be used. If the structure is proportionally damped, equation (4) can be easily uncoupled into a series of SDOF problems. When the structural damping matrix $\mathbf{C}_{\mathrm{ss}}$ of equation (4) is non-proportional, the reduced damping matrix will not be diagonal if the real modes are used in the mode superposition. However, this does not cause any difficulty in the time-domain direct integration, e.g. when using the Newmark method, because the order of the reduced equations of motion is usually quite low. Then an arbitrary pseudo-response $\mathbf{z}$ will be the linear summation of the contributions of all q participant modes, i.e.

$$\mathbf{z} = \sum_{i=1}^{q} \mathbf{z}_i \tag{27}$$

Hence, equation (23) gives the corresponding PSD matrix as

$$\mathbf{S}_{zz}(t, \omega) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mathbf{z}(t, \omega)_i^{*}\,\mathbf{z}(t, \omega)_j^{\mathrm{T}} \tag{28}$$

Clearly, all cross-correlation items between the participant modes have been involved in the PSD matrix of the response $\mathbf{z}$.

STRUCTURAL RESPONSES TO ARBITRARILY COHERENT MULTI-POINT SEISMIC EXCITATIONS

In the above, structures were subjected to multi-phase non-stationary random ground excitations, i.e. the excitations between any two ground joints were fully coherent. In the most general case, excitations are partially coherent. For example, based on the observation data from the well-known SMART-I network, Loh and Yeh[5] proposed the coherency relationship between two arbitrary ground joints as

$$\gamma_{jk} = \gamma(\omega, d) = \exp\left[-\alpha'\left(\frac{\omega|d|}{2\pi v}\right)\right]\exp\left[\mathrm{i}\,\frac{\omega d}{v}\right] \tag{29}$$

in which it was proposed[5] that the constant α' should take the value of 0·125, v represents the speed of the seismic waves and $d = x_k - x_j$ is the distance between the jth and kth ground joints. Previously, Harichandran and Vanmarcke[6] proposed the more complex coherency relationships

$$\left.\begin{aligned}\gamma_{jk} &= \rho(\omega, d) = \rho'(\omega, d)\exp(-\mathrm{i}\omega d/v)\\ \rho'(\omega, d) &= A\exp\left[-\frac{2d}{\alpha\theta(\omega)}(1 - A + \alpha A)\right] + (1 - A)\exp\left[-\frac{2d}{\theta(\omega)}(1 - A + \alpha A)\right]\\ \theta(\omega) &= k[1 + (\omega/\omega_0)^b]^{-0\cdot 5}\end{aligned}\right\} \tag{30}$$

in which, the empirical constants are $A = 0{\cdot}736$, $\alpha = 0{\cdot}147$, $k = 5210$, $\omega_0 = 6{\cdot}85$ and $b = 2{\cdot}78$.

The coherence matrix

$$\mathbf{R} = \begin{bmatrix} 1 & R_{12} & \cdots & R_{1N} \\ R_{21} & 1 & \cdots & R_{2N} \\ & \cdots & \cdots & \\ R_{N1} & R_{N2} & & 1 \end{bmatrix} \tag{31}$$

is given for Loh and Yeh's model, using equation (29), by

$$R_{jk} = \exp(-\alpha\omega|d|/2\pi v) \tag{32}$$

whereas for the Harichandran and Vanmarcke model

$$R_{jk} = \rho'(\omega, d), \quad d = |x_j - x_k| \tag{33}$$

Obviously, $\mathbf{R}$ is a symmetrical real matrix and so its eigenpairs α_j and $\boldsymbol{\psi}_j$ $(j = 1, 2, \ldots, N)$ are easily found. They satisfy

$$\mathbf{R}\boldsymbol{\psi}_j = \alpha_j\boldsymbol{\psi}_j, \quad \boldsymbol{\psi}_j^{\mathrm{T}}\boldsymbol{\psi}_j = \delta_{ij} \qquad (i, j = 1, 2, \ldots, N) \tag{34}$$

in which δ_{ij} is the Kronecker delta. Provided that the matrix $\mathbf{R}$ has r non-zero eigenvalues ($r \leqslant N$), it can be expressed as[9,13]

$$\mathbf{R} = \sum_{j=1}^{r} \alpha_j\boldsymbol{\psi}_j\boldsymbol{\psi}_j^{\mathrm{T}} \tag{35}$$

By repeating the derivation of the last section, equations (9)–(12) still apply, but equation (13) is replaced by

$$\begin{aligned}\boldsymbol{\phi}_{kl}(\tau_k, \tau_l) &= \boldsymbol{\phi}_{kl}(\tau)\\ &= \int_{-\infty}^{\infty}\begin{bmatrix} 1 & R_{12}\exp(\mathrm{i}\omega(T_1 - T_2)) & \cdots & R_{1N}\exp(\mathrm{i}\omega(T_1 - T_N)) \\ R_{21}\exp(\mathrm{i}\omega(T_2 - T_1)) & 1 & \cdots & R_{2N}\exp(\mathrm{i}\omega(T_2 - T_N)) \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1}\exp(\mathrm{i}\omega(T_N - T_1)) & R_{N2}\exp(\mathrm{i}\omega(T_N - T_2)) & \cdots & 1 \end{bmatrix} \mathrm{e}^{\mathrm{i}\omega\tau} S_{xx}(\omega)\,\mathrm{d}\omega\\ &= \int_{-\infty}^{\infty} \mathbf{V}^*\mathbf{R}\mathbf{V}^{\mathrm{T}}\exp(\mathrm{i}\omega\tau)S_{xx}(\omega)\,\mathrm{d}\omega\end{aligned} \tag{36}$$

Substituting equation (35) into equation (36) gives

$$\begin{aligned}\boldsymbol{\phi}_{kl}(\tau_k, \tau_l) = \boldsymbol{\phi}_{kl}(\tau) &= E[\mathbf{X}(\tau_k)\mathbf{X}(\tau_l)^{\mathrm{T}}]\\ &= \int_{-\infty}^{\infty}\sum_{j=1}^{r} \mathbf{V}^*\boldsymbol{\psi}_j\boldsymbol{\psi}_j^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\exp(\mathrm{i}\omega\tau)\alpha_j S_{xx}(\omega)\,\mathrm{d}\omega\end{aligned} \tag{37}$$

Then substituting equation (37) into equation (11) and noting that $\tau = \tau_l - \tau_k$ gives

$$\mathbf{R}_{z_k z_l}(t_k, t_l) = E[\mathbf{z}_k(t_k)\mathbf{z}_l(t_l)^{\mathrm{T}}]$$
$$= \int_0^{t_k}\int_0^{t_l} \mathbf{h}_k(t_k - \tau_k)\mathbf{G}(\tau_k)\int_{-\infty}^{\infty}\sum_{j=1}^{r} \mathbf{V}^*\boldsymbol{\psi}_j\boldsymbol{\psi}_j^{\mathrm{T}}\mathbf{V}^{\mathrm{T}}\exp(\mathrm{i}\omega(\tau_l - \tau_k))\alpha_j S_{xx}(\omega)\,\mathrm{d}\omega$$
$$\times\,\mathbf{G}(\tau_l)^{\mathrm{T}}\mathbf{h}_l(t_l - \tau_l)^{\mathrm{T}}\,\mathrm{d}\tau_k\,\mathrm{d}\tau_l = \sum_{j=1}^{r}\int_{-\infty}^{\infty}\mathbf{I}_{kj}^*\mathbf{I}_{lj}^{\mathrm{T}}\alpha_j S_{xx}(\omega)\,\mathrm{d}\omega \tag{38}$$

in which, where $n = k$ or l,

$$\mathbf{I}_{nj} = \int_0^{t_n}\mathbf{h}_n(t_n - \tau_n)\mathbf{G}(\tau_n)\mathbf{V}\boldsymbol{\psi}_j\exp(\mathrm{i}\omega\tau_n)\,\mathrm{d}\tau_j \tag{39}$$

Clearly, $\mathbf{I}_{kj}$ is the kth structural response at time $t = t_k$ due to the excitation $\mathbf{G}(t)\mathbf{V}\boldsymbol{\psi}_j\exp(\mathrm{i}\omega t)$. Therefore, the corresponding pseudo-excitation can be assumed as

$$\ddot{\mathbf{X}}_{\mathrm{m}j} = \sqrt{\alpha_j S_{xx}(\omega)}\,\mathbf{G}(t)\mathbf{V}\boldsymbol{\psi}_j\exp(\mathrm{i}\omega t) \tag{40}$$

The responses due to this pseudo-excitation component would be

$$\mathbf{z}_{kj} = \sqrt{\alpha_j S_{xx}(\omega)}\,\mathbf{I}_{kj}, \qquad \mathbf{z}_{lj} = \sqrt{\alpha_j S_{xx}(\omega)}\,\mathbf{I}_{lj} \tag{41}$$

From equation (38), the cross-PSD between $\mathbf{z}_k$ and $\mathbf{z}_l$ is

$$\mathbf{S}_{z_k z_l}(t_k, t_l, \omega) = \sum_{j=1}^{r}\mathbf{I}_{kj}^*\mathbf{I}_{lj}^{\mathrm{T}}\alpha_j S_{xx}(\omega) \tag{42}$$

Comparing equations (41) and (42) gives

$$\mathbf{S}_{z_k z_l}(t_k, t_l, \omega) = \sum_{j=1}^{r}\mathbf{z}_{kj}^*\mathbf{z}_{lj}^{\mathrm{T}} \tag{43}$$

It can also be readily justified that the PSD of $\ddot{\mathbf{X}}_{\mathrm{m}}$ is

$$\mathbf{S}_{\ddot{\mathbf{X}}_{\mathrm{m}}\ddot{\mathbf{X}}_{\mathrm{m}}}(t, \omega) = \sum_{j=1}^{r}\ddot{\mathbf{X}}_{\mathrm{m}j}^*\ddot{\mathbf{X}}_{\mathrm{m}j}^{\mathrm{T}} = \sum_{j=1}^{r}\alpha_j S_{xx}\mathbf{G}\mathbf{V}^*\boldsymbol{\psi}_j\boldsymbol{\psi}_j^{\mathrm{T}}\mathbf{V}\mathbf{G} = S_{xx}\mathbf{G}\mathbf{V}^*\mathbf{R}\mathbf{V}\mathbf{G} \tag{44}$$

The above analysis shows that if $\mathbf{R}$ is of rank r, the action of the multi-point ground excitations can be regarded as the linear superposition of r independent random excitation sources $\ddot{\mathbf{X}}_{\mathrm{m}j}$, $j = 1, 2, \ldots, r$. Based on these pseudo-ground accelerations, any required structural responses, denoted as $\mathbf{y}_j$, $\mathbf{z}_j$, etc., can be found easily by means of a step-by-step integration scheme. For large systems, equation (4) can be reduced by using the mode superposition scheme. When the structure is proportionally damped, equation (4) can be completely uncoupled, and the resulting problem is very easy to solve. When the structure is not proportionally damped, the reduced equations of motion will be coupled. However, this does not cause any difficulty in the time domain step-by-step integration, because the order of the reduced set of equations of motion is usually very low. Finally, the global PSD matrices $\mathbf{S}_{yy}$, $\mathbf{S}_{zz}$ and $\mathbf{S}_{yz}$ are given by the left-hand sides of

$$\sum_{j=1}^{r}\mathbf{y}_j^*\mathbf{y}_j^{\mathrm{T}} = \mathbf{S}_{yy}, \qquad \sum_{j=1}^{r}\mathbf{z}_j^*\mathbf{z}_j^{\mathrm{T}} = \mathbf{S}_{zz}, \qquad \sum_{j=1}^{r}\mathbf{y}_j^*\mathbf{z}_j^{\mathrm{T}} = \mathbf{S}_{yz} \tag{45}$$

Because r is generally much smaller than the order of equation (4), using equations (45) is considerably more efficient than direct use of equation (4). In particular, for the fully coherent case given in the previous section, $r = 1$, and the second expression of equation (45) reduces to equation (23).

EXAMPLES

In the examples, the following four seismic excitation models A–D have been used:

A = multi-phase ground motion (=fully coherent asynchronous excitations);
B = uniform ground motion (=synchronous excitations);
C = Loh and Yeh's model (implying partially coherent asynchronous excitations);
D = Harichandran and Vanmarcke's model (also implying partially coherent asynchronous excitations).

Example 1. Bar with two masses

Figure 1 shows a massless uniform bar of length $3L = 3 \times 10$ m with two masses, $M_1 = 5 \times 10^3$ kg and $M_2 = 10^4$ kg, attached to it. The axial rigidity of the bar is $EA = 4 \times 10^3$ kN. The first two angular axial natural frequencies of the system are 7·122 and 13·758 (1/s) and the corresponding damping ratios are both assumed to be 0·05. The evolutionary random ground acceleration has the form of equation (8) with the PSD of the stationary component $x(t)$ being the band-limited filtered white noise

$$S_{xx}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} S_0 \tag{46}$$

with $S_0 = 10^{-2}$ m^2/s^3 within the region $\omega \in [4{\cdot}0, 18{\cdot}0]$ (1/s), $\omega_g = 15{\cdot}46$ (1/s) and $\zeta_g = 0{\cdot}623$, while the modulation function $g(t)$ is

$$g(t) = \begin{cases} 0{\cdot}0 & \text{when } t < 0 \\ 2{\cdot}5974\{\exp(-0.2t) - \exp(-0{\cdot}6t)\} & \text{when } t \geqslant 0 \end{cases} \tag{47}$$

When computing the variances of the chosen responses, the frequency interval used was $\Delta\omega = 0{\cdot}2$ (1/s). The Newmark scheme was used with[14] $\alpha = 0{\cdot}25$, $\delta = 0{\cdot}5$ and $\Delta t = 0{\cdot}01$ (s) when computing the discrete responses of the structure in the time period $t \in [0{\cdot}0, 16{\cdot}0]$ (s). The apparent horizontal pressure wave speed used is 200 m/s, corresponding to very soft soil.

Although this example is very small, it enables the main characteristics of solutions to be observed from Figures 2(a)–2(d) which show the time-dependent PSD distributions of the axial force in the second (middle) bar given by models A–D.

Firstly, for all models, the PSD surfaces have prominent peaks near the first circular natural frequency of 7·122 (1/s).

Secondly, if the structure had been symmetric, the PSD surface for model B would not have had a peak near the second natural circular frequency of 13·758 (1/s), because the anti-symmetric seismic excitation would contribute nothing to a symmetric mode. In fact, the example has a symmetric stiffness distribution but its mass distribution is not symmetric, and so Figure 2(b) has a small peak near the second natural frequency. However, the corresponding peaks are much higher in Figures 2(a), (c) and (d) because, even if the structure had been entirely symmetric, such peaks would still have occurred for the asynchronous ground excitations of models A, C and D. (This phenomenon was also discussed in References 3 and 10).

The third observation is that the PSD surface for model B approaches zero near $\omega = 2$ (1/s), unlike the surfaces given by models A, C and D, which have substantial values near this frequency. Such substantial values are caused by the pseudo-static effect; see equations (26) and (3). In fact, when ω is very small, $\mathbf{X}_m$ and

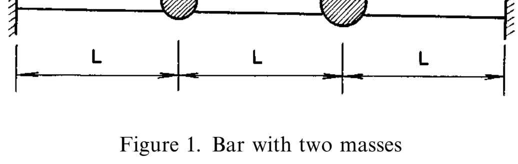

Figure 1. Bar with two masses

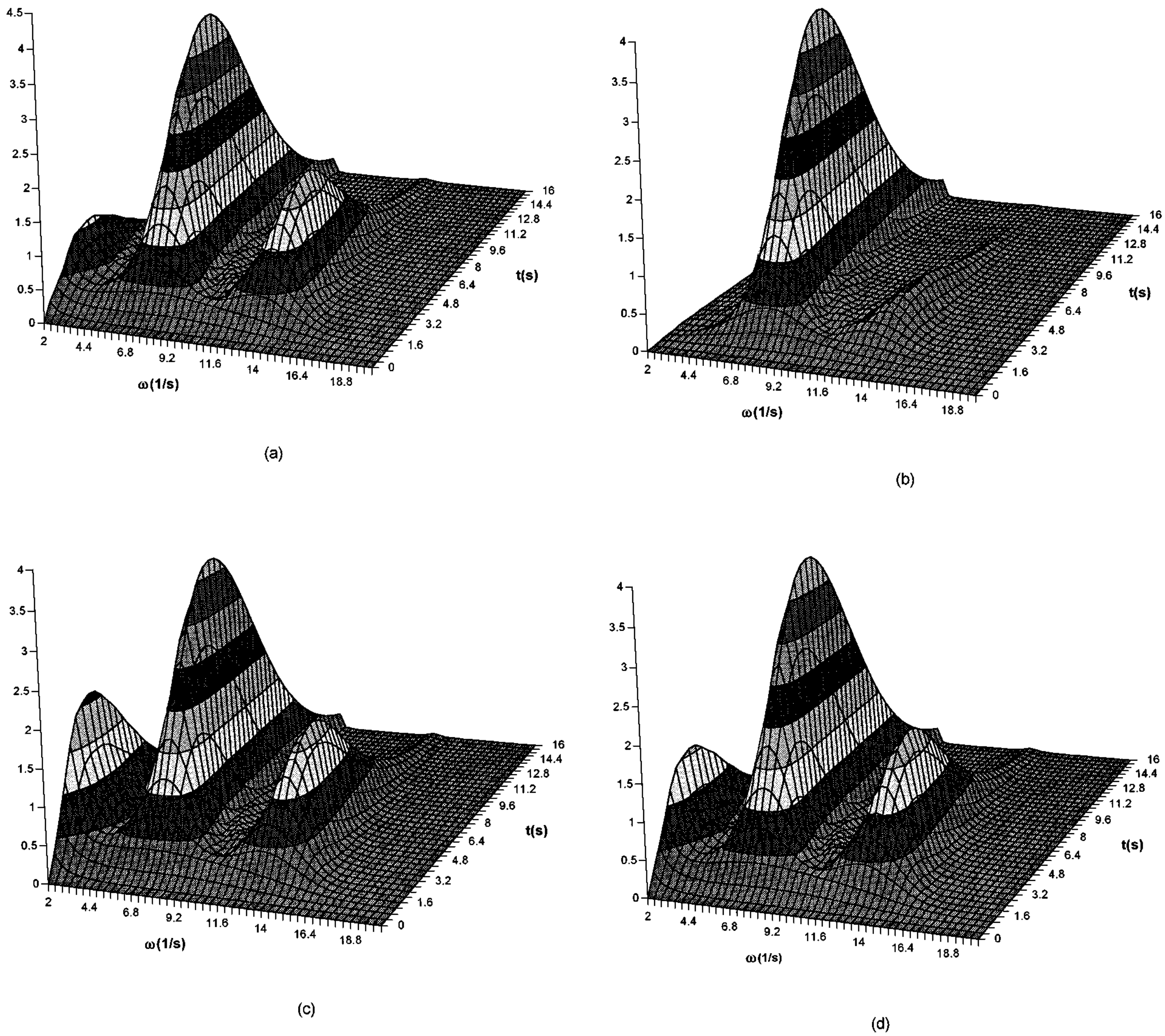

Figure 2. PSD distribution of the axial force of bar 2 for models A–D in units of kN^2s: (a) model A; (b) model B; (c) model C; (d) model D

$\mathbf{Y}_s$ will be very big. In addition, it seems that the loss of coherency between ground joint excitations might be why the substantial values at $\omega = 2$ (1/s) for models C and D are larger than for model A.

Example 2. Four-bay RC plane frame

Figure 3 shows a four-bay reinforced concrete plane frame of total span 4×40 m and of height 15 m at the middle and 10 m at both ends. The cross-section of each column is $1{\cdot}5$ m $\times$ $1{\cdot}5$ m and that of each beam is $1{\cdot}5$ m $\times$ $1{\cdot}0$ m, with the larger dimension being in the plane of the figure. The density is 2500 kg/m^3 and Young's modulus is $1{\cdot}962 \times 10^7$ kN/m^2. Seismic vertical shear (SV) waves were assumed to have an apparent horizontal speed of 300 m/s and can be regarded as an evolutionary random process with the form of equation (8), with the modulation function

$$g(t) = \begin{cases} 0{\cdot}0 & \text{when } t < 0 \\ 1{\cdot}0 & \text{when } t \geqslant 0 \end{cases} \tag{48}$$

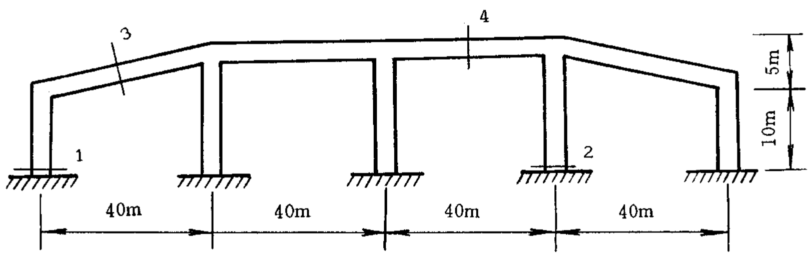

Figure 3. Four-bay plane frame

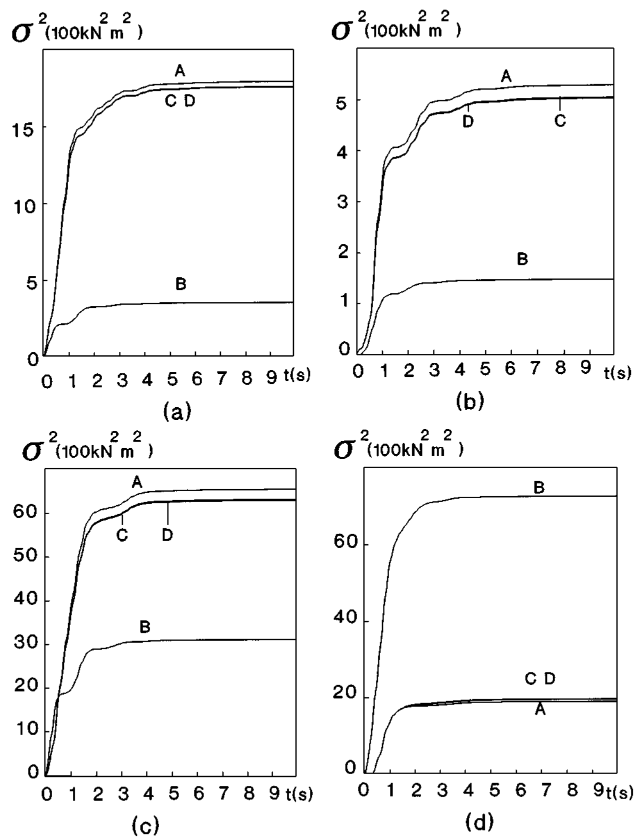

Figure 4. Time-dependent variance distribution of M_y for models A, B, C and D at (a) section 1, (b) section 2, (c) section 3, (d) section 4

and with the PSD of $x(t)$ being a band-limited white noise with $S_0 = 10^{-4}\,m^2/s^3$ within the region $\omega \in [4{\cdot}0, 18{\cdot}0]$ (1/s). All numerical integrations used $\Delta\omega = 0{\cdot}2$ 1/s and $\Delta t = 0{\cdot}01$ s and the Newmark method was used (with $\alpha = 0{\cdot}5$ and $\delta = 0{\cdot}25$) in the time region $t \in [0, 10]$ s.

For this example, five lumped masses were added at the five column tops, respectively, so it has ten dynamic degrees of freedom. The first five in-plane modes were adopted for mode superposition, their corresponding angular natural frequencies being 7·230, 9·927, 10·854, 13·332 and 15·392 (1/s). The damping ratio for each of these participating modes was taken as 0·05.

Figures 4(a)–4(d) give the time-dependent variance curves of the in-plane moments M_y at Sections 1–4.

CONCLUSIONS

The main purpose of this paper is to propose a highly efficient pseudo-excitation algorithm for analysing various non-stationary random responses of multi-support structures subjected to seismic evolutionary random excitations. The numerical examples show that the uniform ground motion model B can lead to quite different results from those of models A, C and D, while results from model A seem to be very close to those from the more elaborate models C and D. However, it should be noted that the parameters used in this paper (such as the cut-off frequencies of the excitations, the apparent seismic wave speeds, the adopted participating modes, etc.) may not be completely appropriate, while the PSD functions used (white noise or filtered white noise) may lead to unreasonable errors near $\omega = 0$ and so must eventually be improved. In addition, the effect of longer structural spans on the loss of coherency should be further investigated. When more reasonable parameters are ascertained, and better PSD models are used, a more comprehensive comparison of such seismic excitation models can be undertaken by using the proposed algorithm.

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